Firm Dynamics in the Neoclassical Growth Model*

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Abstract: This paper integrates firm dynamics theory into the Neoclassical growth framework. It embeds selection into an otherwise standard model of one good, two production factors (capital and labor) and competitive markets. Selection relies on firm specific investment: i) capital is a fixed production factor –an entry cost, ii) the productivity of capital is firm specific, but observed after investment, iii) firm specific capital is partially reversible –its opportunity cost plays the same role as fixed production costs. At equilibrium, aggregate technology is Neoclassical, but TFP is endogenous and positively related to selection; capital depreciation positively depends on selection too, due to capital irreversibility. The Neoclassical model is the limit case of homogeneous firms. At steady state, output per capita and welfare both raise with selection. Rendering capital more reversible or increasing the variance of the idiosyncratic shock both raise selection, productivity, output per capita and welfare.

Keywords: Firm dynamics, Selection, Neoclassical Growth model, Scrapping, Capital irreversibility.

JEL classification numbers: O3, O4.

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1 Introduction

The Neoclassical growth model is a cornerstone of economic theory; modern macroeconomics is founded on it. We can trace its origins to the contributions of Robert Solow [12] and Frank Ramsey [11], among many others. Its power relies on its ability to replicate in a simple and stylized framework the salient evidence on economic growth, the well-known Kaldor [8] stylized facts. Among its main simplifying assumptions, the Neoclassical growth model builds on the assumption of a representative firm.

During the last decades, indeed, following the seminal contributions of Hopenhayn [6] and Jovanovic [7], a new framework has emerged designed to understand the implications of a growing evidence on firms and market behavior. In this literature, selection plays a fundamental role in explaining market performance. Firms are heterogeneous, and competition creates and destroys firms and jobs, moving resources from low productive to high productive firms; the exit-entry process and the expansion-contraction of incumbent firms governs market behavior raising average productivity and output.

In the spirit of both the Neoclassical growth theory and the theory of firm dynamics, this paper puts forward a competitive general equilibrium model with heterogeneous firms. We will refer to it as the Ramsey-Hopenhayn model. Firms’ technology is assumed to use a fixed heterogeneous production factor (capital) and a flexible homogeneous factor (labor) to produce a sole good, which as usual may be consumed or invested. To produce, firms have to pay an entry cost: the price of the fixed factor. The productivity of firms, more precisely the productivity of firms’ capital, is unknown to firms before entry. For the sake of simplicity, as in Melitz [10], we assume that the productivity of firms is time invariant, i.e., firms face no other productivity shock that the initial one. In addition to that, capital is assumed to be partially irreversible. When a firm decides to close down, its capital cannot be directly used for any other firm, but it has, indeed, a scrap value, which is smaller than the replacement value of capital. The fact that capital has an opportunity cost makes firms with low productive capital to exit. Consequently, fixed

1See Bartelsman and Doms [1] for a revision of this literature. The paper also relates to the literature on occupational choice that follows the seminal contribution of Lucas [9]. In line with the survey in Hopenhayn [5], the framework could be extended to study different types of distortions that affect the efficient allocation of productive resources across firms.
production costs are not required to generate endogenous exit at equilibrium.\textsuperscript{2}

At equilibrium, the aggregate technology is Neoclassical with total factor productivity being positively related to selection.\textsuperscript{3} The Ramsey-Hopenhayn model shares then most properties of the Neoclassical theory, but selection critically affects the state of technology. Moreover, capital irreversibility makes selection costly. Indeed, changes in the environment that promote selection are shown to increase steady state welfare. In top of that, the model encompasses the Neoclassical model in the limit cases of homogeneous firms and/or fully reversible capital.

Sections 2 and 3 study the Ramsey-Hopenhayn model under the assumption that there is no technical change. Section 4 extends it to the case of disembodied technical progress.

\section{The Ramsey-Hopenhayn Model}

The model in this paper is a competitive dynamic general equilibrium model with endogenous entry and exit. As in Hopenhayn [6], a continuum of heterogeneous firms produce a homogeneous good under perfect competition. Firm’s technology employs a fixed production factor (capital) and a flexible production factor (labor). Capital is partially irreversible, in the sense that when a firm close down it only recovers a fraction $\theta \in (0, 1)$ of its capital, i.e. its scrap value. Technology has decreasing returns to labor. Firm’s productivity is randomly drawn after entry and it remains constant over time.

There is a measure $N_t$ of identical households, growing at the rate $n$, $n > 0$. Each household member offers inelasticity one unit of labor at any time $t$. The representative household maximizes utility

$$U = \int_0^\infty u(c_t) e^{(n-\rho)t} \, dt$$

subject to a standard budget constraint. Instantaneous utility $u(c_t)$ is Neoclassical, where $c_t$ represents per capita consumption. The subjective discount rate $\rho > n$. Households fully diversify firm specific risks by buying the market portfolio. Consequently, the

\textsuperscript{2}We borrow this assumption from Gabler and Licandro [3].

\textsuperscript{3}The aggregation result is in line with aggregation in the vintage capital literature as in Solow [13] and Solow et al [14].
optimal behavior of the representative household is given by the Euler equation

\[
\frac{\dot{c}_t}{c_t} = \sigma_t(r_t - \rho),
\]  
(2)

where \(\sigma_t\) is the intertemporal elasticity of substitution. At steady state \(r_t = \rho\).

Each firm requires one unit of capital to produce and is characterized by a firm-specific productivity \(z\). Output of a \(z\)-firm is given by

\[
y = A z^{\alpha} \ell^{1-\alpha},
\]  
(3)

where the state of productivity \(A\) is strictly positive. In the next, \(y(z)\) and \(\ell(z)\) denote equilibrium output and employment, respectively, of a firm with productivity \(z\). We show in Annex A that the arguments below are not restricted to the particular case of a Cobb-Douglas production function, but they apply to a general Neoclassical technology.

The entry cost is the cost of buying one unit of capital. To fix ideas, let us interpret the firm specific productivity \(z\) as being embodied in firm’s capital and revealed after investment. In facts, after entry, productivity \(z\) is drawn from the continuous density \(\varphi(z)\), for \(z\) in the support \(Z \in \mathbb{R}^+\) –the cumulative distribution is denoted by \(\Phi(z)\). Expected productivity at entry is assumed to be one without any loss of generality. Firms may exit for two different reasons. First, endogenously, when its value is smaller than the scrap value of capital \(\theta\). Second, firms also exit at the exogenous rate \(\delta\), \(\delta > 0\), in which case the scrap value of capital is zero. Under these conditions, as it will be shown below, there exists a strictly positive productivity cutoff \(z^* \in Z\), such that the distribution of firms at the stationary equilibrium, as in Melitz (2003), is the truncated density

\[
\Phi(z) = \frac{\varphi(z)}{1 - \Phi(z^*)}
\]  
(4)

for \(z \geq z^*\).

3 Stationary Equilibrium

A firm with productivity \(z\) solves

\[
\max_{\ell} A z^{\alpha} \ell^{1-\alpha} - w\ell,
\]
for a given wage rate $w$. Its optimal labor demand is

$$\ell(z) = \left( \frac{(1 - \alpha)A}{w} \right)^{\frac{1}{\alpha}} z.$$

The labor market clearing condition is

$$k \int_{z \geq z^*} \ell(z)\phi(z)d(z) = 1,$$

where $k$ is at the same time the mass of operative firms per capita and aggregate capital per capita (remind that a firm requires one unit of capital to produce). After substitution of individual labor demands $\ell(z)$ into the market clearing condition, the equilibrium wage rate becomes

$$w = \frac{(1 - \alpha)A(\bar{z}k)\alpha}{(average) \text{ marginal productivity of labor}}$$

where $\bar{z}$ measures the average productivity of capital

$$\bar{z} = \int_{z \geq z^*} z\phi(z)dz.$$

When needed, we will make explicit that average productivity depends on the cutoff productivity by denoting it $\bar{z}(z^*)$. It is easy to see that $\bar{z}'(z^*) > 0$; selection raises average productivity, making aggregate technology to be more productive at equilibrium.

Notice that, after substituting for equilibrium wages, labor demand becomes

$$\ell(z) = \left( \frac{1}{k} \right) \frac{z}{\bar{z}}.$$

The first term on the rhs, $1/k$, represents average labor per firm. Moreover, labor is distributed across firms according to the relative productivity $z/\bar{z}$. This is one dimension of the reallocation effect: high productive firms employ more workers than low productive firms.

Firm specific equilibrium profits are then linear on $z$

$$\pi(z) = (\alpha A \bar{z}k^{\alpha-1}) \frac{z}{\bar{z}}.$$

Profits $\pi(z)$ are the return to firm’s capital, which at equilibrium is equal to the marginal product of capital of the average firm multiplied for the firm specific relative productivity.

The value of the $z$-firm at steady state is

$$v(z) = \begin{cases} 
\pi(z) / (\rho + \delta) & \text{if } z \geq z^* \\
\theta & \text{otherwise},
\end{cases}$$

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which, for \( z \geq z^* \), is monotonically increasing in \( z \). Consequently, the exit threshold \( z^* \) is determined by the exit condition

\[
\pi(z^*) = \theta(\rho + \delta) \quad \Rightarrow \quad z^* = \frac{\theta(\rho + \delta)}{\alpha A} (\bar{z}k)^{1-\alpha}.
\]  

(AE)

A firm with productivity \( z < z^* \) exits, since the value of producing is smaller than the scrap value of capital. Exit is then the result of capital not being productive enough.

The free entry condition is

\[
\Phi(z^*) \theta + (1 - \Phi(z^*)) \pi(\bar{z})/(\rho + \delta) = 1.
\]  

(FE)

With probability \( \Phi(z^*) \) the newly created firm draws a productivity smaller than \( z^* \), in which case it closes down and recovers the scrap value of capital. Otherwise, with probability \( 1 - \Phi(z^*) \) it becomes operative. In the last case, its value is equal to the expected value of operative firms.

**Neoclassical Model.** The Neoclassical model corresponds to the particular case of a degenerate distribution \( \Phi(z) \) with zero variance. In this case, there is a representative firm with productivity equal to one. The equilibrium wage is \( w = (1 - \alpha)Ak^\alpha \), corresponding to the marginal product of labor. Profits are \( \pi = \alpha Ak^{\alpha-1} \), which the free entry condition makes equal to \( \rho + \delta \). Indeed, profits are the return to capital, being equal to the marginal product of capital. Notice that irreversibility plays no role: the firms’ value is larger than the scrap value \( \theta \), making all firms optimally stay in the market irrespective of how large \( \theta \) is.

**Selection.** The main results of this paper are shown below. Proposition 1 shows that the Ramsey-Hopenhayn model admits one and only one stationary solution for \( z^* \). Proposition 2 shows that \( z^* \) is an increasing function of \( \theta \), increasing more than proportionally; selection raises with capital reversibility. Since \( z^*/\theta \) affects positively both steady state output and consumption per capita, Proposition 2 also implies that more selective economies produce and consume more in per capita terms. Finally, Proposition 3 shows that a large variance of the productivity distribution is associated with more selection, which increases both stationary output and consumption per capita. As a consequence, the Ramsey-Hopenhayn model shows unambiguous welfare gain from
Combining (EC) and (FE), the equilibrium cut-off is determined by
\[
\frac{z^*}{\theta} = \frac{1 - \Phi(z^*)}{1 - \theta \Phi(z^*)} \bar{z}(z^*) < \bar{z}(z^*).
\] (EC-FE)

Equation (EC-FE) defines \( z^* \) as an implicit function of \( \theta \), say \( z^*(\theta) \), the functional form depending on the functional form of the entry distribution \( \Phi(.) \). (EC-FE) is an arbitrage condition: A firm exits if the expected return of reentering is larger than the current return. For this reason, \( z^* < \theta \bar{z} \). In the Neoclassical model, even under full reversibility of capital, firms never exit since expected productivity is equal to current productivity.\(^5\)

**Proposition 1** For \( \theta \in (\zeta, 1) \), there exists one and only one \( z^* \) that solves (EC-FE)

**Proof**: Multiply both sides of (EC-FE) by \( \theta (1 - \theta \Phi(z^*)) \), move the term \( \theta z^* \Phi(z^*) \) to the right-hand-side and use the definition of \( \bar{z} \) to rewrite the equilibrium condition as
\[
\frac{z^*}{\theta} = \int_{z \geq z^*} z \varphi(z) dz + z^* \Phi(z^*) \equiv A(z^*). \quad (EC-FE')
\]

Let the support of the entry distribution be \((\zeta, \omega)\), \( \zeta \geq 0 \) and \( \omega \leq \infty \). The left-hand-side of (EC-FE’) is linear, crosses the origin and has a slope \( 1/\theta > 1 \). Concerning the right-hand-side, \( A(\zeta) = 1 \) (remind that the entry distribution has unit mean) and \( \lim_{x \to \omega} A(x)/x = 1 \), which is strictly smaller than \( 1/\theta \). Under the assumption that \( \theta > \zeta \), it is easy to see that \( A(x) \) crosses \( x/\theta \) at least once in the interior of \((\zeta, \omega)\).

The first derivative of \( A(x) \) is
\[
A'(x) = \Phi(x) \in (0, 1).
\]

Since the slope of \( A(x) \) is smaller than the slope of the left-hand-side for all \( x > \zeta \), \( A(x) \) can only crosses \( x/\theta \) once, which completes the proof.\( \square \)

\(^4\)In the standard firm selection model, selection is positively related to an increase in fixed production costs. But, an environment with larger fixed production costs is less efficient. Nevertheless, in the Ramsey-Hopenhayn model selection is positively related to capital reversibility, with the polar property that an environment with larger capital reversibility is more efficient.

\(^5\)Notice that in the limit case of \( \theta = 1 \), if the support of the distribution is bounded above by \( \omega < \infty \), \( z^* = \bar{z} = \omega \). Firms will keep trying until the point in which they get the maximum productivity \( \omega \).
In the particular case of a lognormal distribution with unit mean (the variance is set to 0.284), and a degree of irreversibility $\theta = 0.75$, Figure 1 represents both sides of the equilibrium condition (EC-FC), the diagonal (dashed line) and the equilibrium value of $z^*$.  

It is easy to see that there is no selection in an economy with full capital irreversibility. When the distribution admits a lower bound productivity $\zeta = 0$ and $\theta = 0$, the solution with full irreversibility is $z^*(0) = 0$. In the case of Figure 1, the $x/\theta$ line collapses to the y-axis and $z^*$ moves to the point $(0, 1)$. Otherwise, when the distribution admits $\zeta > 0$ and $\theta$ is at its minimum admissible value $\zeta$, $A(x) = x/\theta$ only admits $z^* = \zeta$ as a solution. On the other extreme, when $\theta$ goes to one, $z^*$ goes to $\omega$. The $x/\theta$ locus in Figure 1 moves towards the diagonal cutting the $A(x)$ locus at $\omega$. In case $\omega$ is unbounded, $z^*$ tends to infinity. When capital is fully reversible, firms would always like to try again and again in order to beat the market. As a consequence, the market becomes tougher and tougher, making both $z^*$ and $\bar{z}$ go to infinity.

Let us now prove the following corollary that will be useful for the next.

**Corollary 1** $z^* \geq \theta$

**Proof:** Notice that $z^* = \theta A(z^*)$. Since $A(\zeta) = 1$ and $A'(x) > 0$, then $A(x) \geq 1$ for $x \geq \zeta$. Consequently, $z^* \geq \theta$. $\square$

Corollary 1 can be easily seen in Figure 1, since $A(x)$ is always larger than one, in particular in the equilibrium point, implying that $z^* > \theta$. 

Figure 1: Determination of the cutoff productivity $z^*$
The fact that the cutoff productivity is increasing in $\theta$ can be easily observed in Figure 1. Notice that an increase in $\theta$ moves the $x/\theta$ locus to the right, raising $z^*$. Indeed, as shown in the proposition below, the raise in $z^*$ is more than proportional.

**Proposition 2** \( \frac{dz^*}{d\theta} > 1 \)

**Proof:** Totally differentiate the equilibrium condition $x = \theta A(x)$, use the result above that $A'(x) = \Phi(x)$ and reorganize terms to get

\[
\frac{dx}{d\theta} = \frac{A(x)}{1 - \theta \Phi(x)}.
\]

Notice that the right-hand-side is larger than one, since $A(x)$ is larger than one and $1 - \theta \Phi(x)$ is smaller than one, which completes the proof.\(\Box\)

What is the role of idiosyncratic uncertainty in the selection process? The follow proposition shows that selection is increasing in the dispersion of productivity across firms.

**Proposition 3** *If the support of the initial distribution is bounded from above, $z^*$ is increasing in the variance of the entry distribution*

**Proof:** Let the support of the entry distribution be $(\zeta, \omega)$, $\zeta \geq 0$ and $\omega < \infty$. Firstly, integrate by parts the integral at the right-hand-side of (EC-FE), and multiply both sides by $\theta$, to get

\[
z^* = \theta \left( \omega - \int_{\zeta}^{\omega} \Phi(z)dz \right).
\]

(5)

An equilibrium is a fixed point of this relation (see Proposition 1).

Let the initial distribution be a mean preserving spread (remind that the mean of the entry distribution is supposed to be one) and make it explicit by writing it as $\Phi(z; \sigma)$, where $\sigma$ is the standard deviation.

Let us now consider two distributions of the same family that have different standard deviations $\sigma_1 < \sigma_2$. In this case, for any $z^* \in (\zeta, \omega)$,\(^6\)

\[
\int_{\zeta}^{z^*} (\Phi(z; \sigma_2) - \Phi(z; \sigma_1))dz \geq 0, \quad \Rightarrow \quad \int_{z^*}^{\omega} (\Phi(z; \sigma_2) - \Phi(z; \sigma_1))dz \leq 0.
\]

Consequently, the right-hand-side of (5) will move to the right when $\sigma$ moves from $\sigma_1$ to $\sigma_2$, increasing $z^*$.\(\Box\)

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\(^6\)See Diamond and Stiglitz (1974).
Aggregate Effects of Selection. At equilibrium, from (EC), capital per capita in efficiency units can be written as

$$\bar{z}k = \left(\frac{\alpha A}{\rho + \delta}\right)^{\frac{1}{1-\alpha}} \left(\frac{z^*}{\theta}\right)^{\frac{1}{1-\alpha}}.$$  \hspace{1cm} (k)

It is easy to see that in the limit case of the Neoclassical model, when the variance of the entry distribution converges to zero, $\bar{z} = 1$ and $k = \left(\frac{\alpha A}{\rho + \delta}\right)^{\frac{1}{1-\alpha}}$. Selection makes $\bar{z} > 1$ and adds the term $z^*/\theta$ to the rhs of equation (k). From Corollary 1, $\bar{z}k$ is larger than in the Neoclassical economy, and from Proposition 2 it is increasing in $\theta$. Moreover, since idiosyncratic uncertainty promotes selection, by inducing entry and making survival tougher, it makes capital per capita in efficiency units to be larger.

Aggregate output per capita is

$$y = k \int_{z \geq z^*} A z^\alpha \ell(z)^{1-\alpha} \phi(z) \, dz,$$

where, as said above, total capital per capita in physical units $k$ is equal to the total number of firms. It is easy to see that at equilibrium aggregate technology in per capita terms becomes

$$y = A(\bar{z}k)^\alpha.$$  \hspace{1cm} (y)

After substituting the equilibrium value of $\bar{z}k$ from equation (k), output per capita becomes

$$y = \left(\frac{\alpha A}{\rho + \delta}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{z^*}{\theta}\right)^{\frac{\alpha}{1-\alpha}}.$$  \hspace{1cm} (y)

Since $z^* > \theta$, selection makes output per capita to be larger than in the Neoclassical model –where output per capita is just $k^\alpha$. More important, output per capita is increasing with selection. Idiosyncratic uncertainty opens new opportunities that require some degree of capital reversibility to be realized.

Scrapping. Let $e$ be the mass of entrants per capita. The mass of operative firms per capita evolves following

$$\dot{k} = e \left(1 - \Phi(z^*)\right) - \delta k.$$

Remind that entrants with productivity smaller than $z^*$ automatically exit. At steady state, $e = \frac{\delta}{1 - \Phi(z^*)} k > k$. Since aggregate capital is equal to the mass of firms, selection makes entry larger than the physical depreciation of capital.
In terms of production
\[ y = c + e(1 - \theta \Phi(z^*)) , \]
since the scarp capital \( \theta \) of the \( \Phi(z^*) \) e exiting firms is reemployed in production. Indeed, the usual accounting identity holds
\[ y = c + i , \]
implying that investment per capita
\[ i = e(1 - \theta \Phi(z^*)) . \]
It is easy to see that \( e(1 - \Phi(z^*)) < i < e \). Due to partial reversibility, investment is smaller than gross entry but larger than net entry.

Let us denote the stock of capital in efficiency units by \( \hat{k} = \bar{z} k \). Combining the previous equations and using the (EC-FE) condition, we can easily show that
\[ \dot{\hat{k}} = \frac{z^*}{\theta} i - (\delta + gz) \bar{z} k , \]
where \( g_z \) is the growth rate of \( z^* \), which is nil at steady state. Remind that partial reversibility makes \( z^* < \theta \bar{z} \). The value of investment, \( z^*/\theta \), is given by the exit margin and it is smaller than the average value of existing capital \( \bar{z} \). At steady state
\[ i = \frac{\theta}{z^*} \delta \bar{z} k = \delta \left( \frac{\alpha A}{\rho + \delta} \right)^\frac{1}{1-\alpha} \left( \frac{z^*}{\theta} \right)^\frac{\alpha}{1-\alpha} . \]
Investment per capita is larger than in the Neoclassical economy, growing with both \( \theta \) and \( \sigma \).

**Welfare.** Finally, it is easy to show that per capita consumption at equilibrium is
\[ c = y - i = \left( \left( \frac{\alpha A}{\rho + \delta} \right)^\frac{\alpha}{1-\alpha} - \delta \left( \frac{\alpha A}{\rho + \delta} \right)^\frac{1}{1-\alpha} \right) \left( \frac{z^*}{\theta} \right)^\frac{\alpha}{1-\alpha} . \]  
\[ \text{Neoclassical} \]
Due to selection, stationary consumption per capita, and then welfare, are also larger than in the Neoclassical economy. It is important to notice that scrapping is related to capital irreversibility. More efficient economies, those with larger \( \theta \), are more selective, have a larger total factor productivity (as measured by \( \bar{z} \)), produce and consume more. For the same argument than before, steady state welfare is also larger in economies with a more disperse productivity distribution at entry.
4 Technical Progress

The model can be easily extended to the case of exogenous technical progress by assuming that technical progress is disembodied affecting both new and old capital. For this purpose, let us make the following assumptions:

1. The productivity of incumbent firms grow at the exogenous rate \( \frac{1-\alpha}{\alpha} \gamma \), \( \gamma > 0 \).

2. At any time \( t \), new firms draw a productivity \( z_t \) from the stationary continuous density \( \varphi(z_t e^{-\frac{1-\alpha}{\alpha} \gamma t}) \), for \( z_t \) in the support \( 0 \leq \zeta e^{-\frac{1-\alpha}{\alpha} \gamma t} < \omega e^{-\frac{1-\alpha}{\alpha} \gamma t} \leq \infty \). The density \( \varphi(.) \) is assumed to have expected value equal to one at time \( t = 0 \). The expected value then grows at the rate \( \frac{1-\alpha}{\alpha} \gamma \).

Under these assumptions, the equilibrium distribution is the truncated entry distribution, truncated at \( z_t e^{-\frac{1-\alpha}{\alpha} \gamma t} = z^* \). At a balance growth path, \( y, c, k \) and \( w \) will be all growing at the rate \( \gamma \). From the Euler equation (2), \( r = \rho + \gamma / \sigma \), where \( \sigma \) is the intertemporal elasticity of substitution, that we assume constant for simplicity. This will slightly modify the (EC) and (FE) conditions, since profits \( \pi(z_t) \) and the value of the firm \( v(z_t) \) remain time invariant. In facts, in both conditions the term \( \rho + \delta \) has to be substituted by \( \rho + \gamma / \sigma + \delta \). Indeed, the condition (EC-FE) becomes

\[
\frac{z^*_t}{\theta} = \frac{1 - \Phi(z^*_t e^{-\frac{1-\alpha}{\alpha} \gamma t})}{1 - \theta \Phi(z^*_t e^{-\frac{1-\alpha}{\alpha} \gamma t})} \tilde{z}(z^*_t).
\]

Multiplying both sides for \( e^{-\frac{1-\alpha}{\alpha} \gamma t} \) and making the variable change \( z^* = z^*_t e^{-\frac{1-\alpha}{\alpha} \gamma t} \), we get again equation (EC-FE) above. Notice that \( \tilde{z}(z^*_t) = \tilde{z}(z^*) e^{\frac{1-\alpha}{\alpha} \gamma t} \). Since (EC-FE) remains unchanged, the equilibrium cutoff \( z^*_t = z^* e^{\frac{1-\alpha}{\alpha} \gamma t} \). This will also affect \( \tilde{k}, y \) and \( c \) in the Neoclassical model, and through it the equilibrium of the Ramsey-Hopenhayn model.

5 Endogenous Growth

As shown in the Appendix, the argument applies to any firm’s technology \( x = F(z, \ell) \), increasing in and homogeneous of first degree on both arguments \( z \) and \( \ell \), and concave on \( \ell \). It then applies, in particular, to firm’s technology

\[
x = Az + z^\alpha \ell^{1-\alpha},
\]
which aggregates at equilibrium on the aggregate technology

\[ y = A \bar{z}k + (\bar{z}k)^{\alpha}. \]

Selection, as in the Ramsey-Hopenhayn model, only depends on the scrap value of capital \( \theta \) and the parameters of the entry density productivity distribution \( \varphi(z) \). For a given average productivity \( \bar{z} \), the economy behaves as in Jones and Manuelli (...). At the unique balance growth path, under the assumption that household preferences are constant intertemporal elasticity of substitution, the economy grows at the endogenous rate

\[ g = \sigma(A\bar{z}^{\alpha} - \rho - \delta). \]

Selection by making the productivity of capital larger, the term \( A\bar{z}^{\alpha} \), positively affects the growth rate of the economy. [CHECK IT]

6 Conclusions

This paper extends the Neoclassical growth model to accommodate for firm heterogeneity in the spirit of the literature on firm dynamics. In this framework, selection raises total factor productivity and per capita output. More important, it increases steady state welfare.

The argument also applies to endogenous growth models of the AK type, like Jones and Manuelli (...). In this case, selection has a positive effect on the endogenous growth rate by increasing the average productivity of capital.

References


A General Technology

A firm with productivity $z$ solves

$$\max_\ell F(z, \ell) - w\ell,$$

for a given wage rate $w$. Technology $F(z, \ell)$ is assumed to be increasing in and homogeneous of first degree on both arguments $z$ and $\ell$, and concave on $\ell$. The optimal labor demand is

$$F_2(z, \ell) = w \Rightarrow \ell(z) = G^{-1}(w)z,$$

where $G(\ell/z) = F_2(1, \ell/z)$. Remind that $F(.)$ is homogeneous of first degree, then first derivatives are homogeneous of degree zero. Since $F_{22}(z, \ell) < 0$, $G'(.) < 0$.

The labor market clearing condition is

$$k \int_{z^*}^{\omega} \ell(z)\phi(z)d(z) = 1.$$

After substitution of individual labor demands $\ell(z)$ into the market clearing condition,

$$k \ G^{-1}(w) \int_{z^*}^{\omega} z\phi(z)d(z) = 1.$$

The equilibrium wage rate is

$$w = G(1/(\bar{z}k)),$$

where $\bar{z}$ is the average productivity of capital, defined as in the main text. Since $G'(.) < 0$, the wages rate is positively related to aggregate capital measured in efficiency units.

Substituting the equilibrium wage rate into the labor demand function, it becomes

$$\ell(z) = \frac{z}{\bar{z}k},$$

as in the main text.

Firm specific equilibrium profits are linear on $z$

$$\pi(z) = F(1, 1/(\bar{z}k))z.$$ 

It is easy to see that the (EC-FE) condition do not depend on the particular form of the production function. Consequently, equation (EC-FE) holds, implying that there exists a unique equilibrium cutoff $z^*$ (Proposition 1), increasing with but faster than $\theta$. 

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(Proposition 2). Indeed, the (EC) condition gives the equilibrium value of capital in efficiency units

$$\bar{z}k = \left( F^{-1} \left( \frac{\theta (\rho + \delta)}{z^*} \right) \right)^{-1}. $$

Capital in efficiency units is growing with selection, in particular if selection results from an increase in the degree of capital reversibility $\theta$.

Per capita output results from aggregation of firms’ production:

$$y = F(\bar{z}k, 1).$$

Notice that the aggregate technology has the same functional form as the individual technology (remind that aggregate labor is one). Since output follows capital in efficiency units, production increases with selection, in particular if it results from an increase in capital reversibility.